Varianta 92

SUBIECTUL I

a) $AB = 2\sqrt{2}$.
b) $S_{ABC} = 6$.
c) $\sin^2 30^\circ + \cos^2 30^\circ = 1$.
d) $\bar{z} = 3 - 4i$.
e) $S = 4$
f) $a = 1$.

SUBIECTUL II
1.
a) $m_a = 15$.
b) 5% din 40 este: $\frac{5}{100} \cdot 40 = 2$.
c) p=$\frac{3}{10}$.
d) $\log_2 \frac{1}{4} = -2$.
e) Restul este $f(1) = 2$.
2.
a) $f(1) = 0$.
b) $x_1 = 0,\ x_2 = -1,\ x_3 = 1$.
c) $f'(x) = 3x^2 - 1$.
d) $\int_0^1 f(x)dx = -\frac{1}{4}$.
e) $\lim_{x \to \infty} \frac{f(x)}{x^3} = 1$.

SUBIECTUL III

a) $\det(A) = 1$.
b) $A^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$.
c) $A^2 - 2A + I_2 = O_2$ **.**

d) Fie $P(n): A^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$, n∈**N*.**

$P(1): A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ (A).

Presupunem $P(k)$ (A) şi demonstrăm $P(k+1)$ (A), unde k≥1.

$P(k): A^k = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}$, iar $P(k+1): A^{k+1} = \begin{pmatrix} 1 & 2k+2 \\ 0 & 1 \end{pmatrix}$.

Dar $A^{k+1} = A^k \cdot A \overset{P(k)}{=} \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2k+2 \\ 0 & 1 \end{pmatrix}$**.**

De unde $P(n)$ este (A) ∀n∈**N*.**

e) $(A - I_2)^2 = (A - I_2)(A - I_2) = A^2 - 2A + I_2 = O_2$**.**

f) $\det A + \det(A^2) + \ldots + \det(A^{2007})$=2007, deoarece ∀n∈**N*,** $\det A^n = \begin{vmatrix} 1 & 2n \\ 0 & 1 \end{vmatrix} = 1$.

g) $\det(A + A^2 + \ldots + A^{2007}) = \begin{vmatrix} 2007 & 2(1+2+\ldots+2007) \\ 0 & 2007 \end{vmatrix}$ =2007^2.

SUBIECTUL IV

a) $f(1) = \frac{1}{2}$.

b) Calculăm expresia: $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)} = f(x), \forall x > 0$**.**

c) $f'(x) = \left(\frac{1}{x^2 + x}\right)' = \frac{-2x-1}{(x^2+x)^2}$, $x > 0$.

d) Deoarece $f'(x) = \frac{-2x-1}{(x^2+x)^2} < 0, \forall x > 0 \Rightarrow$ funcția f este descrescătoare pe $(0, \infty)$.

e) $a_n = f(1) + f(2) + \ldots + f(1) \overset{b)}{=} \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots + \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$.

f) $\lim\limits_{n\to\infty} n(a_n - 1) = \lim\limits_{n\to\infty} \frac{-n}{n+1} = -1$.

g) Calculăm $\int_n^{n+1} f(x)dx \overset{b)}{=} \int_n^{n+1} \left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \ln x\Big|_n^{n+1} - ln(x+1)\Big|_n^{n+1} = \ln\frac{(n+1)^2}{n(n+2)}$.

De unde $\lim\limits_{n\to\infty} \int_n^{n+1} f(x)dx$ =0.